THE ROLE OF COMPLIANT FINGERPADS IN GRASPING AND MANIPULATION: IDENTIFICATION AND CONTROL*

ANURADHA M. ANNASWAMY[†] AND MANDAYAM A. SRINIVASAN[†]

Abstract. Manual exploration and manipulation of unknown objects in unstructured environments require sensory guided motor control strategies. For humans or general purpose robots, the presence of compliant fingerpads is crucial in enhancing the stability of grasp and manipulability, and the objects they encounter are often compliant. In this paper, we apply well known system identification and control methods to enable successful grasping and manipulation of compliant objects using compliant fingerpads. Through the use of linear and nonlinear lumped parameter models, we describe the dynamic relationships between the external forces exerted on the fingers and the contact forces imposed on the object. We present two approaches to realize the necessary control actions, one where the identification of the system parameters is followed by control, and the other where an adaptive control strategy is used. We illustrate the importance of tactile information in not only satisfying the necessary interface constraints, but also in simplifying the identification and control procedures for successful performance of grasping and manipulation tasks.

1. Introduction. Haptics, which pertains to manual exploration and manipulation of objects in an environment, is important to both humans and robots. A detailed and quantitative understanding of the underlying dynamics, information flow, and control strategies will benefit investigations of human haptics and development of robots. It is especially valuable in the development of haptic interfaces through which humans can interact manually with teleoperated systems or computer generated virtual environments. Although the principles of operation of man-made devices are quite different from those of humans, the constraints on the performance of these haptic tasks, such as the laws of physics governing the mechanics of contact and the presence of friction and gravity are the same for both. In addition, the types of tactual sensory information, their processing and the computation of the required control actions are sufficiently similar for the two systems that the common aspects of information processing can be functionally separated from the hardware implementations that carry it out. Therefore, a theory that investigates what kinds of information are necessary, and how they have to be processed in order to successfully complete a desired haptic task can be common to humans, robotic systems, and dynamic interactions between the two. In this paper, we take the first steps towards analyzing the identification and control issues that arise during grasping and manipulation of unknown compliant objects with compliant fingerpads, albeit in a simplified context.

Almost all haptic tasks can be classified as exploration, manipulation,

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† Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139.

or a combination of the two. The goal of haptic exploration is to extract information about the surface (for example, shape, surface texture) and material properties (for example, mass, compliance) of objects in the environment. The goal of manipulation is to alter the environment in a desired manner and requires either knowledge of the relevant object properties through prior exploration, or on-line adaptation of the control strategies based on the behavior of the object while it is being manipulated. In analyzing haptics, it is therefore critical to investigate the computations underlying the identification of task parameters and the control of tasks. In order to develop a computational theory of haptics along the lines advocated by Marr [1] in his work on computations in the visual system, it is necessary to first have a set of 'competence' theories, i.e., simplified theories that address what the system might be trying to do and how it could be doing it [2]. Ultimately, the competence theories may evolve to a 'performance' theory that is specific to humans or robots in explaining the actual operation of the system of interest. The analysis of grasping and manipulation developed in this paper is to be viewed as a simple competence theory that ignores many of the complexities (such as those due to spatial variations of forces within contact regions and nonlinearities in the mechanics of contact) for the sake of mathematical tractability in focusing on identification and control aspects. With suitable extensions, such an analysis helps in improving the performance of autonomous robots, generating hypotheses for human haptics, and designing of haptic interfaces.

In the literature on robotics, both the robot end effectors and the objects in contact with them are generally assumed to be rigid (see [3,4,5,6,7])) The resulting mechanics of contact gives rise to simple mathematical models and enables, in theory, direct control of contact forces which govern the performance of the task. However, these algorithmic advantages have to be traded off against serious disadvantages in the actual performance of the tasks with respect to force equilibrium, grasp stability, and control of contact forces. Since point contact has no torsional resistance (i.e., to rotations about normal to the object surface at contact locations), higher number of rigid fingers are needed for equilibrium in certain grasp configurations, than if the fingers are deformable. Also, point contact is not stable with respect to rotations about axes tangential to the object surface at the contact point, and is highly sensitive to local aberrations of the contacting surfaces. For rigid-rigid contacts, the friction coefficient is generally much lower than if any of the contacting entities is deformable, thus requiring the normal forces to be generally larger for a stable grasp, especially in a gravity environment. Since these higher forces are also concentrated at isolated point contacts, they can crush or break fragile objects.

Finite contact region, whether due to object compliance, robot fingerpad compliance, or both, overcomes many of the disadvantages of point contact: increased resistance and better grasp stability with respect to rotations of the object, reduction of undesirable sensitivity to local aberrations of contacting surfaces, increased friction coefficient and hence reduced normal forces distributed over a finite contact area. However, because compliant fingerpads act effectively as passive deformable links intervening between the object and the actively controlled rigid support of the finger (such as the bone in humans), direct control of contact forces is not possible. The interaction between the fingerpad and the object becomes complex, since the forces of interaction are dependent on the dynamic parameters such as mass, damping, and elastic stiffness of each of the contacting entities, as well as the interface parameters such as friction. Suitable dynamic models of interaction and appropriate information processing are essential for successful execution of tasks, and the lumped parameter models used here represent an initial step.

In contrast to the best available robots, humans seem to perform dextrous manipulation of objects effortlessly with their hands, even when the mechanical properties of the object are unknown *a priori*. This ability is predicated upon proper integration of the mechanical, sensory, motor, and cognitive subsystems that constitute the human haptic system. The structure of the fingers consists mainly of compliant tissues supported by relatively rigid bones. The compliant tissues are passive and the motions of the bones are actively controlled by the muscles, with the control action ranging from a fast spinal reflex to a relatively slow conscious action. The skin, joints, tendons, and muscles are richly innervated by a wide variety of mechanosensitive receptors that convey sensory information to the brain through associated nerve fibers [8]. This tactual sensory information can be divided into two categories: (1) *Tactile*, which refers to direct information about the contact conditions at the interface, such as normal and shear forces, existence and direction of slip [9], etc; (2) *kinesthetic*, which refers to the positions and motions of, and the forces acting on, the bones, conveyed not only by the receptors in the skin around the joints, within joints, tendons, and muscles, but also by information derived from motor commands for intended movements. When the fingerpads come in contact with an object, their compliance generally ensures that the contact region has a finite area, and hence rich information contained in the spatio-temporal variation of the mechanical loads is conveyed to the receptors and subsequently to the brain. A detailed analysis of the dynamics and control in a particular task, even with idealized models such as those employed in this paper, provides hypotheses to be tested in experiments with human subjects so as to understand how the human haptic system works.

The development of haptic interfaces is a relatively new research area where the identification and control methods discussed here are useful. Haptic interfaces are robotic 'master' devices that enable a human user to manually interact with computer generated virtual environments or teleoperated 'slave' robots. Detailed analysis of the dynamic interactions between the human user and real objects as well as those between the 'slave' robot and its environment is necessary to (1) design haptic interfaces for virtual position and relative velocity. As seen in Fig. 1, each finger is assumed to consist of n masses, and the object to have $2p$ masses.

At the contact interface, the fingers exert forces f_{c_l} and f_{c_r} on the object. Since compressive forces are assumed to be positive, it follows that the object stays in contact with the fingers if these two forces are positive. Hence, when in contact, $f_{c_l} > 0$ and $f_{c_r} > 0$ and

$$x_{f_n} = x_{o_1} \quad \text{and} \quad x_{o_{2p}} = x_{f_{n+1}}. \tag{2.3}$$

and when not in contact, $f_{c_l} = f_{c_r} = 0$. Assuming that this constraint is satisfied, the dynamic model in Eq. (1a)-(1i) can be simplified further. Also, for the object to stay in grasp without slip while being manipulated in a gravity environment,

$$f_{c_l} \geq \frac{Mg}{2\mu}, \qquad f_{c_r} \geq \frac{Mg}{2\mu} \tag{2.4}$$

where M is the total mass of the object and μ is the friction coefficient at the contact interface. Another constraint on the force is to prevent the object from being crushed. This implies that

$$f_{c_l} \leq f_{\text{crush}} \qquad f_{c_r} \leq f_{\text{crush}}. \tag{2.5}$$

When both the fingers are in contact with the object, Eqs. (1c) and (1d) as well as Eqs. (1f) and (1g) can be combined to form

$$\begin{aligned} M_n \ddot{x}_{f_n} + \lambda_{o_1}(\dot{x}_{c_{f_n}}, x_{c_{f_n}}) - \lambda_{f_{n-1}}(\dot{x}_{c_{f_{n-1}}}, x_{c_{f_{n-1}}}) &= 0 \\ M_{n+2p-1}\ddot{x}_{f_{n+1}} + \lambda_{f_{n+1}}(\dot{x}_{c_{f_{n+1}}}, x_{c_{f_{n+1}}}) - \lambda_{o_{2p-1}}(\dot{x}_{c_{o_{2p-1}}}, x_{c_{o_{2p-1}}}) &= 0 \end{aligned}$$

where $M_n = m_{f_n} + m_{o_1}$ and $M_{n+2p-1} = m_{f_{n+1}} + m_{o_{2p}}$. Defining

$$\begin{aligned} M_i &= m_{f_i} & i &= 1,..,(n-1) \\ M_{n+i} &= m_{o_{i+1}} & i &= 1,..,(2p-2) \\ M_{n+2p+i-1} &= m_{f_{n+i+1}} & i &= 1,..,(n-1) \end{aligned}$$

the dynamics given in Eq. (1a)–(1i) of the entire system with both the fingers grasping the object can be reduced to

$$\begin{aligned} \ddot{x}_{c_1} + a_1\lambda_1 - \theta_2\lambda_2 &= f_l\theta_1 \\ \ddot{x}_{c_i} + a_i\lambda_i - \theta_{i+1}\lambda_{i+1} - \theta_i\lambda_{i-1} &= 0 \qquad i = 2,..,2N-2 \\ \ddot{x}_{c_{2N-1}} + a_{2N-1}\lambda_{2N-1} - \theta_{2N-1}\lambda_{2N-2} &= f_r\theta_{2N} \\ \ddot{x}_{2N-1} + \theta_{2N-1}\lambda_{2N-1} - \theta_{2N-1}\lambda_{2N-2} &= 0 \end{aligned} \tag{2.6}$$

where $N = n + p - 1$, $\theta_i = (1/M_i)$, $a_i = \theta_i + \theta_{i+1}$, and M_i correspond to the finger masses for $i = 1,..,n-1, n+2p,..,2N$, and to the object masses

for $n+1 \leq i \leq n+2p-2$. The variable x_i corresponds to the motion of mass M_i, x_{c_i} corresponds to the relative motion $x_i - x_{i+1}$, and λ_i denotes the force due to the ith spring and damper, $i = 1,..,2N-1$. For the case when the spring as well as the damper elements exhibit linear dynamics,

$$\lambda_i = b_i\, \dot{x}_{c_i} + k_i x_{c_i} \tag{2.7}$$

As seen from Eq. (2.6), the dynamics of the composite system is described by a nonlinear 2-input dynamic model with $2N$ degrees of freedom, where the two inputs are due to the external forces exerted on the left and right fingers, the degrees of freedom are due to the $2m_f$ masses of the fingers and the $2p$ masses of the object. Since we assume that the object always remains in contact with the fingers in deriving Eq. (2.6), the degrees of freedom are reduced to $2N$.

3. Symmetries. For ease of exposition, we introduce some symmetries into the problem. We choose the left and right finger to be identical and that the object is symmetric about its center. Hence,

$$\theta_i = \theta_{2N-i+1} \qquad i = 1,..,N. \tag{3.1}$$

If the springs and dampers have linear characteristics so that Eq. (2.7) holds, we have

$$B_i = B_{2N-i} \qquad K_i = K_{2N-i} \qquad i = 1,..,N. \tag{3.2}$$

As a result, the number of parameters that require to be identified gets significantly reduced. More importantly, the multivariable dynamic model can be decoupled into two single-input systems where the inputs correspond to the symmetric and asymmetric components of the external forces f_l and f_r. This is expressed in Theorem 1.
Theorem 1: Expressing the external inputs f_l and f_r as

$$f_l = f_s + f_a \qquad f_r = f_s - f_a \tag{3.3}$$

if the system is symmetric so that Eqs. (3.1) and (3.2) are valid, Eq. (2.6) can be simplified as

$$\begin{aligned} &\ddot{x}_{c_{s1}} + a_1\lambda_{s1} - \theta_2\lambda_{s2} = f_s\theta_1 \\ &\ddot{x}_{c_{si}} + a_i\lambda_{si} - \theta_{i+1}\lambda_{s(i+1)} - \theta_i\lambda_{s(i-1)} = 0 \qquad i = 2,..,N-2 \\ &\ddot{x}_{c_{s(N-1)}} + a_{N-1}\lambda_{s(N-1)} - \theta_{N-1}\lambda_{s(N-1)} - 2\theta_N\lambda_{sN}(x_N) = 0 \\ &\ddot{x}_{sN} + 2\theta_N\lambda_{sN}(x_{sN}) - \theta_N\lambda_{s(N-1)} = 0 \end{aligned} \tag{3.4}$$

and

$$\begin{aligned} \ddot{x}_{c_{a1}} + a_1\lambda_{a1} - \theta_2\lambda_{a2} &= f_a\theta_1 \\ \ddot{x}_{c_{ai}} + a_i\lambda_{ai} - \theta_{i+1}\lambda_{a(i+1)} - \theta_i\lambda_{a(i-1)} &= 0 \qquad i = 2,..,N-2 \\ \ddot{x}_{c_{a(N-1)}} + a_{N-1}\lambda_{a(N-1)} - \theta_{N-1}\lambda_{a(N-2)} &= 0 \\ \ddot{x}_{aN} - \theta_N\lambda_{a(N-1)} &= 0 \end{aligned} \tag{3.5}$$

where

$$x_{ci} = \begin{cases} x_{c_{si}} + x_{c_{ai}} & i = 1,..,N \\ -x_{c_{si}} + x_{c_{ai}} & i = N+1,..,2N \end{cases}$$

$$x_i = \begin{cases} x_{si} + x_{ai} & i = 1,..,N \\ -x_{si} + x_{ai} & i = N+1,..,2N \end{cases}$$

$$\begin{aligned} \lambda_{si} &= b_i\, \dot{x}_{c_{si}} + k_i x_{c_{si}} & i = 1,..,N-1 \\ \lambda_{sN}(x_{sN}) &= b_N\, \dot{x}_{sN} + k_N x_{sN}, \quad x_{c_sN} = 2x_{sN} & \end{aligned}$$

$$\begin{aligned} \lambda_{ai} &= b_i\, \dot{x}_{c_{ai}} + k_i x_{c_{ai}} & i = 1,..,N-1 \\ x_{c_aN} &= 0. & \end{aligned}$$

Also, the contact forces f_{c_l} and f_{c_r} can be expressed in terms of their symmetric and asymmetric component f_{c_s} and f_{c_a}, respectively, as

$$f_{c_l} = f_{c_s} + f_{c_a} \qquad f_{c_r} = f_{c_s} - f_{c_a}.$$

The proof follows from simple substitution.

4. A simple model. We shall focus our attention on Eqs. (3.4) and (3.5) to develop identification procedures for the object parameters. We shall consider for the most part the case when $n = 2, p = 1$. In this case, each finger as well as the object have two degrees of freedom, so that $N = 2$, and Eqs. (3.4) and (3.5) represent fourth order systems. The input-output relations between forces and displacements can be described as below, when the symmetries in Eqs. (3.1) and (3.2) hold:

$$(4.1a) \quad \frac{x_{c_s1}}{f_s} = \frac{\theta_1(s^2 + 2\theta_2 b_2 s + 2\theta_2 k_2)}{(s^2 + 2\theta_2 b_2 s + 2\theta_2 k_2)(s^2 + a_1 b_1 s + a_1 k_1) - 2\theta_2^2(b_2 s + k_2)(b_1 s + k_1)}$$

$$(4.1b) \quad \frac{x_{s2}}{f_s} = \frac{\theta_2\theta_1(b_1 s + k_1)}{(s^2 + 2\theta_2 b_2 s + 2\theta_2 k_2)(s^2 + a_1 b_1 s + a_1 k_1) - 2\theta_2^2(b_2 s + k_2)(b_1 s + k_1)}$$

$$(4.1c) \quad \frac{x_{c_a1}}{f_a} = \frac{\theta_1}{s^2 + a_1 b_1 s + a_1 k_1}$$

$$(4.1d) \quad \frac{x_{a2}}{f_a} = \frac{\theta_2\theta_1(b_1 s + k_1)}{s^2(s^2 + a_1 b_1 s + a_1 k_1)}$$

For a general n and p, similar transfer functions can be derived, which will all be of order $2N$. Yet another quantity that will feature in our discussions to follow is the contact forces between each finger and the object.

In terms of their symmetric and asymmetric components, the constraints in Eq. (2.4) and (2.5) can be written as

$$(4.2) \qquad f_{\text{crush}} \geq f_{c_s} \geq |f_{c_a}| + \frac{Mg}{2\mu}$$

In addition, with $\theta_{o1} = 1/m_{o1}$, and we can conclude from Eq. (1d) that

$$(4.1e) \quad \frac{x_{s2}}{f_{c_s}} = \frac{\theta_{o1}}{s^2 + 2\theta_{o1} b_2 s + 2\theta_{o1} k_2}$$

$$(4.1f) \quad \frac{x_{a2}}{f_{c_a}} = \frac{\theta_{o1}}{s^2}$$

5. System identification and control. In the previous section, we derived the underlying dynamic model in Eq. (1) and assuming that the object is always held in contact, we simplified the dynamics of the composite system to Eq. (2.6). The introduction of symmetries in the problem and the reduction of the motion to the x-direction led to the input-output relations (4.1a)–(4.1f). We now proceed with the task of identifying the various system parameters to determine the requisite control forces for carrying out grasping and manipulation. These parameters can be classified into three groups, the constraint parameters $\{f_{\text{crush}}, M, \mu\}$ (in Eq. (2.4)), the finger parameters, and the object parameters. We first discuss the constraint parameters and then proceed to identify the parameters of the finger as well as the object, in the case of both a linear model and a nonlinear model. For the sake of clarity, all our discussions are restricted to the case when $n = 2, p = 1$.

It is obvious that in order to carry out either the identification or the control task, the object has to be retained in grasp without slipping or getting crushed. Therefore any contact forces generated must be such that they satisfy Eqs. (2.4) and (2.5). Towards this end, f_{crush}, M, and μ, need to be identified first. f_{crush} is an inherent property of the object, and we shall assume that it is known. M can be determined by applying a grasp force slightly less than f_{crush} and measuring the vertical force needed to hold the object in air. The friction coefficient μ can be identified by using simple quasi-static procedures. By applying to both fingers, a constant grasp force in the x-direction and a ramp force starting from zero in the upward direction until the fingers slip against the object surface, μ can be obtained as simply the ratio of the grasping force and $Mg/2$ at the incipience of slip. It should be noted that this is based on the assumption that the coulomb friction law is valid. Such an assumption may not be valid in general, and more sophisticated friction models may be necessary.

6. Identification of a linear model. When the stiffness and viscosity properties of the finger and the object are linear, the problem reduces simply to parameter identification in a linear system, which can be solved

using standard results in adaptive identification (see [11] for example). We briefly outline the relevant results below:
Result 1 ([11], Chapter 4): Let $\{u(\cdot), y(\cdot)\}$ be a scalar input-output pair related by a stable transfer function $W(s)$ of order n, so that

$$y(t) = W(s)u(t) \tag{6.1}$$

The system in (6.1) can be represented in the form of an algebraic equation given by

$$y(t) = \theta^T \omega(t)$$

where

$$\begin{aligned} \dot{\omega}_1 &= \Lambda\omega_1 + \ell u \\ \dot{\omega}_2 &= \Lambda\omega_2 + \ell y \\ \theta = [\theta_1^T,\, \theta_2^T]^T,\quad \omega &= [\omega_1^T,\, \omega_2^T]^T \\ \Lambda \in \mathbb{R}^n \text{ is stable} \qquad & (\Lambda, \ell) \text{ is completely controllable} \end{aligned}$$

and $\theta \in \mathbb{R}^{2n}$ contains the $2n$ parameters of the transfer function $W(s)$.
Result 2 ([11], Chapter 4): Given the system described by (6.1), an estimate $\widehat{\theta}$ of θ can be determined using the following identifier:

$$\begin{aligned} \dot{\widehat{\omega}}_1 &= \Lambda\widehat{\omega}_1 + \ell u \\ \dot{\widehat{\omega}}_2 &= \Lambda\widehat{\omega}_2 + \ell y \\ \widehat{y} &= \widehat{\theta}_1^T\widehat{\omega}_1 + \widehat{\theta}_2^T\widehat{\omega}_2 \\ \widehat{\theta} &= [\widehat{\theta}_1^T,\, \widehat{\theta}_2^T]^T, \qquad \widehat{\omega} = [\widehat{\omega}_1^T,\, \widehat{\omega}_2^T]^T \\ \dot{\widehat{\theta}} &= -\Gamma\,(\widehat{y} - y)\,\widehat{\omega} \end{aligned} \tag{6.2}$$

where Γ is a symmetric positive definite matrix.
Result 3 ([11], Chapter 2): For the system in (6.1) and the identifier in (6.2), a necessary and sufficient condition for $\widehat{\theta}(t)$ to converge to θ as $t \to \infty$ is that ω satisfy the condition

$$\int_t^{t+T} \omega(\tau)\omega^T(\tau)\, d\tau \;\geq\; \alpha I \;\; \forall\, t \geq t_0 \tag{6.3}$$

where $\alpha, T > 0$.
Result 4 ([11], Chapter 6): ω as well as $\widehat{\omega}$ satisfy condition (4.1) if the input u is chosen to be of the form

$$u(t) = \sum_{i=1}^{n} a_i \sin \omega_i t \tag{6.4}$$

where the ω_i's are distinct, and $a_i \neq 0$ for $i = 1, \ldots, n$.

In the context of the problem under consideration, assuming that the fingers are identical and that the object is symmetric about its center, Eqs. (3.4) and (3.5) describe the underlying dynamics whose input-output representations are given in Eq. (4.1a)–(4.1f). In order to use Results 1–4 for parameter identification, different system variables need to be measured. As mentioned in the introduction, the presence of tactile information implies that the deformations x_{c_1} and x_{c_3} of the finger as well as the contact forces $f_{c_{x_1}}$ and $f_{c_{x_2}}$ can be measured. On the other hand, availability of kinesthetic information implies that the displacements x_1 and x_4 of the finger, and the external forces f_l and f_r can be measured. Results 1–4 imply that the object parameters and the finger parameters can be identified using any one of the transfer functions in Eq. (4.1a)–(4.1f). For instance, if f_l and f_r are such that f_s has four distinct frequencies, the coefficients of the transfer function in Eq. (4.1a) and hence the system parameters can be identified. Care needs to be taken however so that throughout the identification process, the fingers stay in contact with the object.

While these results suffice for the object and parameter identification, we examine the transfer functions in (4.1a)–(4.1f) in more detail so that simpler identification procedures can be developed by making use of the prior information, the variables present, or the kind of external forces applied. In particular, we discuss the different identification procedures that can be developed when equal forces are applied on the left and the right finger.

From the input-output relations in Eq. (4.1a)–(4.1f), it can be seen that both the finger and the object parameters can be identified using a symmetric set of grasping forces. Hence, in the absence of initial conditions, if $f_l = f_r = f_s$, the motions of the left and the right fingers will be equal and opposite, and the relations (13a), (13b), and (13e) suffice for parameter identification. In addition, $f_{c_l} = f_{c_r} = f_{c_s}$. The constraint in (4.2) can then be simplified further as

$$f_{\text{crush}} \;\geq\; f_{c_s} \;\geq\; \frac{Mg}{2\mu}. \tag{6.5}$$

In the following, we apply such equal and symmetric forces and determine conditions on f_s under which the object and finger parameters can be identified.

The identification procedures become simpler as more variables become available for measurement, and with increasing prior information. Concerning the former, we consider two cases: (i) tactile and kinesthetic information available, (ii) only kinesthetic information is available. Concerning prior information, if the finger parameters are known prior to the identification of the object, then simple procedures can be developed. We discuss these issues below.

(i) *With tactile and kinesthetic information:* If both tactile information and kinesthetic information are present, the variables $\{f_{c_s}, x_{c_{s1}}\}$ and

$\{f_s, x_{s1}\}$ can be directly measured. Hence, $x_{s_2} = x_{s1} - x_{c_{s1}}$ can be computed. As a result, the finger dynamics and the object dynamics become decoupled, since the relevant transfer function

$$\text{(1e)} \qquad \frac{x_{s2}}{f_{c_s}} = \frac{\theta_{o1}}{s^2 + 2\theta_{o1} b_2 s + 2\theta_{o1} k_2}$$

is independent of finger parameters. A procedure similar to that in results 1–4 can be used to identify θ_{o1}, b_2, and and k_2 by choosing f_s (and hence f_{c_s}) to have two independent frequencies. Similarly, the finger parameters can be identified by contacting the finger against a known stationary rigid object. In this case, since the object is stationary, $x_{s2} = 0$ and hence, Eq. (1a) becomes

$$\text{(6.6)} \qquad \frac{x_{s1}}{f_s} = \frac{\theta_1}{s^2 + \theta_1 b_1 s + \theta_1 k_1}.$$

It should be noted that in both the above cases, if the velocities $\dot{x}_{s1}$ and $\dot{x}_{s2}$ are also available, the structure of the identifers reduces to the simplest form possible, since this corresponds to the case when all the four states of the underlying system are available for measurement. In addition, when there are perturbations present which introduce deviations in the dynamic behavior of the system from that described by Eq. (2.1), we cannot ensure that the constraints in (3.5) are satisfied unless tactile information is present. This is discussed further in section 3.1.2.

(ii) *When only kinesthetic information is available:* The finger parameters can be identified prior to the manipulation of the object by contacting the finger against a known stationary rigid object as described in (i), or by symmetrically grasping a rigid object with both fingers. Since $x_{s1} = x_{c_{s1}} + x_{s2}$, the transfer function between f_s and x_{s1} can be determined as

$$\frac{x_{s1}}{f_s} = \frac{\theta_1(s^2 + \theta_2(b_1 + 2b_2)s + \theta_2(k_1 + 2k_2)}{(s^2 + 2\theta_2 b_2 s + 2\theta_2 k_2)(s^2 + a_1 b_1 s + a_1 k_1) - 2\theta_2^2(b_2 s + k_2)(b_1 s + k_1)} \qquad \text{(6.7)}$$

Once the finger parameters θ_1, b_1, and k_1 are identified, we note that Eq. (6.7) can be simplified further as

$$\text{(6.8)} \qquad \begin{aligned} z(t) &= W_o(s) x_{s1}(t) \\ \text{where } z(t) &= \left[\frac{s^2 + \theta_1 b_1 s + \theta_1 k_1}{\theta_1 (b_1 s + k_1)^2}\right] x_{s1}(t) - \left[\frac{1}{(b_1 s + k_1)^2}\right] f_s(t) \\ W_o(s) &= \frac{\theta_2}{s^2 + \theta_2(b_1 + 2b_2)s + \theta_2(k_1 + 2k_2)} \end{aligned}$$

Since $z(t)$ and $x_{s1}(t)$ can be measured on line, the parameters of the second order transfer function $W_o(s)$ can once again be estimated using the same procedure as in (i). Once again, care should be taken to ensure that the constraints are satisfied so that the object is held in stable grasp.

If on the other hand, the finger parameters cannot be identified a priori, and f_s and x_{s1} are the only signals that can be measured, Eq. (6.7) can be used to determine both the finger and object parameters simultaneously. The identifier has the form given below:

$$\text{(6.9)} \qquad \begin{aligned} \omega_1(t) &= \frac{N_1(s)}{Q(s)} f_s(t) \qquad \omega_2(t) = \frac{N_2(s)}{Q(s)} x_{s1}(t) \\ \widehat{x}_{s1} &= \widehat{\theta}_1^T(t)\omega_1 + \widehat{\theta}_2^T(t)\omega_2 \\ \dot{\widehat{\theta}}_1 &= -\Gamma_1(\widehat{x}_{s1} - x_{s1})\omega_1 \\ \dot{\widehat{\theta}}_2 &= -\Gamma_2(\widehat{x}_{s1} - x_{s1})\omega_2 \\ \widehat{\theta} = [\widehat{\theta}_1^T, \widehat{\theta}_2^T]^T & \qquad \omega = [\omega_1^T, \omega_2^T]^T \\ \Gamma_1 \text{ and } \Gamma_2 & \text{ are symmetric positive-definite matrices} \end{aligned}$$

where $Q(s)$ is a Hurwitz polynomial of degree 4 and $N_1(s) \in \mathbb{R}^3$ and $N_2(s) \in \mathbb{R}^4$ have linearly independent elements. Eq. (6.9) corresponds to an adaptive observer structure where we have made use of the fact that $W_s(s)$ has relative degree two and hence only seven parameters have to be identified. From Result 3, it follows that if ω is persistently exciting for all $t \geq t_0$, $\widehat{\theta}(t)$ converges to its true value. This in turn is achieved by choosing $f_s(t)$ to consist of four sinusoids with distinct frequencies.

While, in principle, one can discuss a scenario where only tactile information is present, it is not a realistic one both from a human and robotic point of view. Hence, though simple identification procedures can be developed even for this case, we do not consider it in detail here.

Another mode of external force application that can be used to identify the object corresponds to its asymmetric motion of the object. Since all parameters of the object can be identified during the grasping mode, this mode provides no additional information. Also, the underlying transfer function in this mode, given by Eq. (4.1c), is unstable. Hence, the identifier must include a stabilizing component that is adapted to the system uncertainties. Stable adaptive methods exist which pertain to the identification of such systems [11], and can be applied in this case.

7. Meeting constraints. The above discussions indicate that with equal forces applied to the left and right fingers, provided the nonlinear constraints in Eq. (6.5) are satisfied, standard system identification procedures can be applied to identify the object and finger dynamics. Hence, in addition to satisfying the persistent excitation conditions as in Result 4, f_s must be chosen so that f_{c_s} satisfies Eq. (6.5). Once the object is identified, one can then proceed to formulate the manipulation problem in the workspace and determine the control input needed to realize the objective. The success of the resulting control system is naturally dependent on the fidelity of the dynamic model in (2.1). In reality, there can be several situations where the true system deviates from the model in (2.1). The causes of such deviations include bounded disturbances, unmodeled (typically high frequency) dynamics due to other neglected degrees of freedom, nonlinearities in the viscosity, and elasticity of the fingerpads or the object, measurement noise, or variations in the parameters of the finger or the object due to operating conditions (such as temperature, orientation, aging).

The various perturbations can introduce two kinds of anomalies, one affecting the satisfaction of constraints, and the other introducing errors in the parameter estimation. Essentially, the perturbations can be viewed as introducing additional forces not included in (2.1). These forces may lead to a violation of the constraints in (6.5). Suppose f_{c_s} falls below the lower bound, detection of slip is of paramount importance, and can be accomplished by specialized tactile slip detectors. In fact, it has been demonstrated that primates use tactile sensors to detect slip [9]. In human studies, it has been shown that subjects apply an external force which is about 30% more than the lower bound needed to overcome slip [12]. When slips occur within the contact region, the grasp forces are increased automatically through reflex action, even without direct attention of the subject when these slips are too small to detect consciously. However, such grasp force adjustments do not take place if tactile information is blocked by cutaneous anesthesia. In the absence of such detectors, if tactile information is available, then f_{c_s} can be measured on-line which in turn can be used to appropriately increase f_s. If on the other hand, only kinesthetic information is available, then f_s can be increased based only on an estimate of f_{c_s}. Hence, under sufficiently large perturbations, it may not be possible to retain the object in stable grasp.

8. Identification of a nonlinear model. When the viscosity and stiffness properties become nonlinear, Eq. (2.6) can be used to describe the dynamics of the resulting composite nonlinear system. Assuming that $N=2$, the equations are given by

$$\begin{aligned}\ddot{x}_{c_1} + a_1\lambda_1 - \theta_2\lambda_2 &= \theta_1 f_l\\ \ddot{x}_{c_2} + a_2\lambda_2 - \theta_3\lambda_3 - \theta_2\lambda_1 &= 0\\ \ddot{x}_{c_3} + a_3\lambda_3 - \theta_3\lambda_2 &= \theta_4 f_r\\ \ddot{x}_3 + \theta_3\lambda_3 - \theta_3\lambda_2 &= 0\end{aligned} \tag{8.1}$$

Assuming that the left and right fingers are identical, $a_i = \theta_i + \theta_{i+1}$, and that the stiffness and viscosity properties are such that

$$\lambda_i(\dot{x}_i, x_i) = -\lambda_i(-\dot{x}_i, -x_i)$$

we can simplify Eq. (8.1) further as

$$\begin{aligned}\ddot{x}_{c_1} + (\theta_1+\theta_2)\lambda_1 - \theta_2\lambda_2 &= \theta_1 f_l\\ \ddot{x}_{c_2} - \theta_2(2\lambda_2 + \lambda_1 + \lambda_3) &= 0\\ \ddot{x}_{c_3} + (\theta_1+\theta_2)\lambda_3 - \theta_2\lambda_2 &= \theta_1 f_r\\ \ddot{x}_3 - \theta_2(\lambda_2 + \lambda_3) &= 0\end{aligned} \tag{8.2}$$

Here, θ_1 corresponds to the mass of the left (or right) finger, while θ_2 corresponds to the total mass of the object and the contacting finger on the left (or the right). x_{c_1} and x_{c_3} correspond to the deformation of the left and the right finger, while x_{c_2} corresponds to the object deformation. x_3 describes the absolute motion of the right finger and hence that of the composite system. All the λ_i's represent the forces due to the ith spring and damper, $i = 1,2,3$, with λ_1 and λ_3 corresponding to those of the finger, and λ_2 to that of the object.

Assuming that λ_i are linear in the unknown parameters for $i = 1,2,3$, the first equation in Eq. (8.2) can be expressed as

$$\begin{aligned}\ddot{x}_{c1} &= \theta_1 f_l - (\theta_1+\theta_2)\lambda_1 + \theta_2\lambda_2\\ &= \theta^{*T}\omega(t)\end{aligned}$$

where θ^* is a vector containing the unknown parameters, and $\omega(t)$ is a vector of signals which depend on $\{f_l, x_{c1}, \dot{x}_{c1}, x_{c_2}, \dot{x}_{c_2}\}$. Hence, if both tactile and kinesthetic information is available, then $\omega(t)$ can be measured at each instant of time. By making use of the fact that the underlying system is of second order, a stable estimator can be constructed for identifying θ^* and is in the following form:

$$\begin{aligned}\ddot{x}_m + c\,\dot{x}_m + kx_m &= \theta^T\omega + c\,\dot{x}_{c1} + kx_{c1}\\ \dot{\theta} &= -\gamma\,(ce + (k+1)\,\dot{e})\,\omega \qquad \gamma > 0\end{aligned} \tag{8.3}$$

where $e = x_m - x_{c1}$, and c and k are arbitrary positive constants. Defining $\bar{e} = [e, \dot{e}]^T$ and $\phi = \theta - \theta^*$, Eq. (8.3) can be rewritten as

$$\dot{\bar{e}} = A\bar{e} + b\phi^T\omega \qquad \dot{\phi} = -\gamma I\bar{e}^T Pb\omega \tag{8.4}$$

where

$$A = \begin{bmatrix} 0 & 1\\ -k & -c\end{bmatrix}, \qquad b = \begin{bmatrix} 0\\ 1\end{bmatrix}, \quad P = \begin{bmatrix} c^2+k^2+k & c\\ c & k+1\end{bmatrix}.$$

Since Eq. (8.4) is in the form commonly used in parameter identification problem (see [11], p. 126), it follows that $\bar{e}(t) \to 0$ as $t \to \infty$. Asymptotic convergence of $\theta(t)$ to θ^* follows if ω satisfies the persistent excitation condition

$$\int_t^{t+T} \omega\omega^T\, d\tau \geq \alpha I, \qquad \forall t \geq t_0, \qquad \alpha > 0,\ T > 0.$$

The unknown parameters in θ^* comprise of quantities related to both the fingers and the object, and in a nonlinear manner. Hence, it would be quite tedious to determine the individual parameters representing the mass, stiffness, and viscosity properties. Prior knowledge of the finger parameters would facilitate the identification of the object parameters. This can be accomplished simply by contacting the finger one at a time against a known rigid object at rest. Also, as discussed in the linear case, through out the identification procedure, we need to ensure that the constraints in Eqs. (2.4) and (2.5) are always satisfied. In the foregoing procedure, we have assumed that the fingers stay in contact with the object. As in the linear case, the presence of tactile information along with slip detectors can be used to increase the grasping force to maintain contact with the object while identification is in progress.

It is worth noting that the estimation scheme outlined above requires the measurement of the vector $\omega(t)$, which in turn requires that both kinesthetic and tactile information be available. As in the linear case, constraints are harder to satisfy in the absence of tactile information. Also, when only kinesthetic information is available, all the states of the dynamic system in (8.2) are not available. Hence, a nonlinear observer needs to be constructed to identify the parameters and may not be possible in general to achieve global identification.

9. Control. Our aim is to move the object along a prescribed path in the workspace while grasping it such that the constraints (2.4) and (2.5) are satisfied. Alternately, the problem can be stated as the control of the system in Eq. (8.2) so that the object position x_3 follows a prescribed trajectory $x^*(t)$ while simultaneously satisfying the constraints (2.4) and (2.5). The system in Eq. (8.2) is an eighth order multivariable nonlinear system with the origin as an equilibrium state. The goal is therefore to find the control inputs f_l and f_r in Eq. (8.2) such that (1) all solutions will be globally bounded for any initial conditions, (2) trajectory following is accomplished, and (3) all other states converge to zero as $t \to \infty$.

When both tactile and kinesthetic information is available, assuming that the object is always held in contact, this problem can be solved in its entirety, and the external control forces can be determined so that stable manipulation is achieved. We do not discuss the solution in detail, but refer the reader to [13]. Also, the solution to the control problem could be considered as a special case of the adaptive control problem discussed at length in the next section.

10. Adaptive control. The discussion in section 5 indicates that with a dynamic model of the composite system as in Eq. (8.2) (for which the linear systems in Eqs. (3.4) and (3.5) are special cases), stable manipulation can be carried out by first identifying the dynamic parameters in Eq. (8.2) and then determine the strategies for generating the external forces for manipulation. Alternately, the tasks of identification and control can be carried out simultaneously using adaptive control strategies which enable the determination of a controller whose parameters are updated on-line using the system measurements, which we shall discuss in this section.

We consider the problem of manipulation of an object when $N = 2$ in the dynamic model in Eq. (1). We assume that

(i) the finger dynamics is known,
(ii) both tactile and kinesthetic information is available, and
(iii) the constraints in Eqs. (2.5) and (2.6) are satisfied at all times.

The cases when the object dynamics is linear and nonlinear are both considered. The aim is to ensure that the object position follows a desired trajectory $x^*(t)$. In addition, we prescribe a bound x_{c2}^* for the deformation of the object so that $|x_{c_2} - x_{c2}^*|$ is required to go to zero as well.

11. The linear model. From Eq. (3.5), it follows that the underlying equations are given by

$$\ddot{x}_{c_{a1}} + (\theta_1+\theta_2)\left[b_1\,\dot{x}_{c_{a1}} + k_1 x_{c_{a1}}\right] = f_{a_1}\theta_1 \tag{11.1}$$

$$\ddot{x}_{a_2} - \theta_2\left[b_1\,\dot{x}_{c_{a1}} + k_1 x_{c_{a1}}\right] = 0 \tag{11.2}$$

We shall assume that the finger dynamics is known so that the parameters θ_1, b_1, and k_1 are known. Eq. (11.1) can therefore be simplified by choosing

$$f_{a_1} = \frac{1}{\theta_1}\left[u + \theta_1(b_1\,\dot{x}_{c_{a1}} + k_1 x_{c_{a1}})\right]$$

Defining $\lambda_1 = b_1\,\dot{x}_{c_{a1}} + k_1 x_{c_{a1}}$, Eqs. (11.1) and (11.2) become

$$\begin{aligned} \ddot{x}_{c_{a1}} + \theta_2\lambda_1 &= u \\ \ddot{x}_{a2} - \theta_2\lambda_1 &= 0. \end{aligned} \tag{11.3}$$

The problem is to determine the control input u in Eq. (11.3) so that the output x_{a2} follows x^* asymptotically.

It is worth pointing out that when the state $X_a = [x_{c_{a1}}, \dot{x}_{c_{a1}}, x_{a2}, \dot{x}_{a2}]^T$ is accessible, this problem cannot be solved using standard adaptive control methods [11]. This is simply due to the structure of the system which is of the form

$$\dot{X}_a = AX_a + bu$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\theta_2 k_1 & -\theta_2 b_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \theta_2 k_1 & \theta_2 b_1 & 0 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

With a feedback controller of the form

$$u = K^T X_a + v,$$

the class of reference models whose states can be followed by X_a are of the form

$$\dot{X}_{ma} = A_m X_{ma} + bv$$

where $A_m = A + bK^T$. The structure of the system matrices A and b implies that A_m cannot be chosen arbitrarily; even if A_m can be made asymptotically stable, the model states X_{ma} cannot be generated since A_m contains the unknown parameter θ_2 of the object.

We therefore proceed to describe an adaptive controller which makes use of the specific structure in the system and ensures asymptotic tracking. This is outlined in Theorem 2.

Theorem 2: For the system in Eq. (11.3), an adaptive controller of the form

$$(11.4)\quad \begin{aligned} u &= \frac{1}{b_1}\left[-ke_{c_{a1}} - T_1 - \widehat{\theta}_2 T_2\right] \qquad k>0 \\ e_{c_{a1}} &= b_1\,\dot{x}_{c_{a1}} + k_1 x_{c_{a1}} + \widehat{p}_1 e_{a_2} + \widehat{p}_2\,\dot{e}_{a_2} + \widehat{p}_3 \ddot{x}^* \qquad e_{a2} = x_{a2} - x^* \\ \dot{\widehat{\theta}}_2 &= e_{c_{a1}} T_2 \\ T_1 &= k_1\,\dot{x}_{c_{a1}} + (\widehat{p}_1 + \dot{\widehat{p}}_2)\,\dot{e}_{a2} + \dot{\widehat{p}}_1\, e_{a2} + (\dot{\widehat{p}}_3 - \widehat{p}_2)\ddot{x}^* + \widehat{p}_3 x^{*(3)} \\ T_2 &= k_v e_{a2} + (k_p+1)\,\dot{e}_{a2} + (\widehat{p}_2 - b_1)\lambda_1 \\ \dot{\widehat{p}}_1 &= (k_v e_{a2} + (k_p+1)\,\dot{e}_{a2})e_{a2} \\ \dot{\widehat{p}}_2 &= (k_v e_{a2} + (k_p+1)\,\dot{e}_{a2})\,\dot{e}_{a2} \\ \dot{\widehat{p}}_3 &= (k_v e_{a2} + (k_p+1)\,\dot{e}_{a2})\ddot{x}^* \end{aligned}$$

ensures that all the signals in the closed-loop system given by Eqs. (11.3) and (11.4) are bounded and $\lim_{t\to\infty}|x_{a2}(t) - x^*(t)| = 0$.

Proof. Let a reference trajectory $x_{c_{r_{a1}}}$ be chosen for $x_{c_{a1}}$, and define $e_{c_{a1}} = x_{c_{r_{a1}}} - x_{c_{a1}}$, and $e_{a2} = x_{a2} - x^*$. We shall choose $x_{c_{r_{a1}}}$ such that
(i) if $e_{c_{a_1}}(t) \to 0$ as $t\to\infty$, then $e_{a2}(t) \to 0$ as $t\to\infty$.
(ii) Then u can be chosen such that $e_{c_{a_1}}(t) \to 0$ as $t\to\infty$.
In the following, we shall show how steps (i) and (ii) can be carried out. Let

$$(11.5)\quad x_{c_{r_{a1}}} = x_{c_{a1}} + \lambda_1 + \widehat{p}_1 e_{a_2} + \widehat{p}_2\,\dot{e}_{a_2} + \widehat{p}_3\ddot{x}^*$$

Then, from Eq. (11.3), we obtain that e_{a_2} satisfies the differential equation

$$\ddot{e}_{a_2} + k_v\,\dot{e}_{a_2} + k_p e_{a_2} = \theta_2\left[e_{c_{a1}} - \widetilde{p}^T W_1\right]$$

where $\widetilde{p} = [\widehat{p}_1 - \frac{k_p}{\theta_2}, \widehat{p}_2 - \frac{k_v}{\theta_2}, \widehat{p}_3 + \frac{1}{\theta_2}]^T$, and $W_1 = [e_{a_2}, \dot{e}_{a_2}, \ddot{x}^*]^T$. It follows that a Lyapunov function candidate of the form

$$V_1 = \frac{1}{2}\left[\overline{e}_{a_2}^T P\overline{e}_{a_2} + |\theta_2|\widetilde{p}^T\widetilde{p} + e_{c_{a_1}}^2\right]$$

has a time-derivative

$$(11.6)\quad \dot{V}_1 = -\overline{e}_{a_2}^T Q\overline{e}_{a_2} + e_{c_{a_1}}[\dot{e}_{c_{a_1}} + \theta_2\overline{e}_{a_2}^T Pb]$$

if the adaptive law for adjusting $\widetilde{p}$ is chosen as

$$\dot{\widetilde{p}} = -sgn(\theta_2)\overline{e}_{a_2}^T PbW_1,$$

where

$$\overline{e}_{a_2} = \begin{bmatrix} e_{a_2} \\ \dot{e}_{a_2}\end{bmatrix},\; P = \begin{bmatrix} k_p^2 + k_v^2 + k_p & k_v \\ k_v & k_p+1\end{bmatrix},\; Q = 2k_pk_vI,\; b = \begin{bmatrix} 0 \\ 1\end{bmatrix}.$$

Since

$$\dot{e}_{c_{a1}} = T_1 + \theta_2(\widehat{p}_2 - b_1)\lambda_1 + b_1 u,$$

Eq. (11.6) can be rewritten as

$$\dot{V}_1 = -\overline{e}_{a_2}^T Q\overline{e}_{a_2} + e_{c_{a_1}}[T_1 + \theta_2 T_2 + b_1 u].$$

Hence a control input of the form

$$u = -\frac{1}{b_1}\left[-ke_{c_{a_1}} - T_1 - \widehat{\theta}_2 T_2\right] \qquad k>0$$

leads to the expression

$$\dot{V}_1 = -\overline{e}_{a_2}^T Q\overline{e}_{a_2} - ke_{c_{a_1}}^2 - \widetilde{\theta}_2 e_{c_{a_1}} T_2$$

where $\widetilde{\theta}_2 = \widehat{\theta}_2 - \theta_2$. Hence, updating V_1 as $V_1 = V_2 + \frac{1}{2}\widetilde{\theta}_2^2$ and adjusting $\widehat{\theta}_2$ as in Eq. (11.4), we obtain that

$$\dot{V}_2 = -\overline{e}_{a_2}^T Q\overline{e}_{a_2} - ke_{c_{a_1}}^2 \le 0.$$

This ensures that the variables $\overline{e}_{a_2}$, $e_{c_{a_1}}$, $\widehat{\theta}_2$, $\widehat{p}_1$, $\widehat{p}_2$, and $\widehat{p}_3$ are bounded. Since the desired trajectory x^* and its first three derivatives are bounded, it follows from the choice of $e_{c_{a_1}}$ that λ_1 is bounded. Since

$$\dot{x}_{c_{a_1}} + \frac{k_1}{b_1}x_{c_{a_1}} = \frac{1}{b_1}\lambda_1,$$

it follows that $x_{c_{a_1}}$ is the output of a first-order system with a bounded input and hence is bounded. This in turn implies that x_{a2} is bounded, which establishes the boundedness of all the state variables of the closed-loop system. Barbalat's lemma and the form of the derivatives further ensures that

$$\lim_{t\to\infty} e_{c_{a_1}}(t) = 0 \qquad \lim_{t\to\infty}\overline{e}_{a2}(t) = 0$$

which concludes the proof. □

As mentioned in the beginning of this section, when assumptions (i)-(iii) were satisfied, the adaptive manipulation problem of an unknown object can be solved. A similar approach can be used in the absence of assumption (i) by developing a controller for the system in Eqs. (11.1) and (11.2). When assumption (ii) is not valid, the input-output representation in Eq. (4.1c) can be used to develop an adaptive controller using standard results in adaptive control [11]. In all the above procedures, it is assumed that the contact force always satisfies

$$f_{\text{crush}} \geq f_{c_s} \geq \frac{Mg}{2\mu} \tag{6.5}$$

Since f_{c_a} is determined by the control input in Theorem 2, to ensure that contact is retained, sufficient grasping force f_{c_s} is present which can be ensured by increasing the force f_s. If the magnitude of f_{crush} is not large enough to tolerate large increases in f_s, it implies that large asymmetric forces f_{c_a} cannot be tolerated. In such circumstances, f_{a_1} and hence u must be constrained to lie within a certain magnitude. The adaptive control problem then can be posed as follows:

Let the input-output representation be given by

$$x_{a_1} = W(s)u$$

where $W_p(s)$ has unknown coefficients and the control input $u(t)$ is restricted to lie within a magnitude of $u_{\max}$ for all t. The problem is to find u so that x_{a_1} follows a desired trajectory reasonably closely. This problem was addressed in [14] where it was shown that when the initial conditions on the plant-state and the control parameters are small, stability of the closed-loop system and trajectory following with a small error is possible.

12. The nonlinear model. We consider the control of the system in Eq. (8.2), which is an eighth order multivariable nonlinear system by first studying its stabilization. From the description of the dynamic model in section 2, it follows that the origin is an equilibrium state of the system in Eq. (8.2). The aim here is to find the control inputs f_l and f_r in Eq. (8.2) such that all solutions will be globally bounded for any initial conditions and asymptotically converge to the origin as $t \to \infty$. As in the linear case, it is assumed that (i) the finger dynamics is known, i.e., θ_1, and the functions λ_1 and λ_3 are known, and that θ_2 and the function λ_1 are unknown, (ii) tactile and kinesthetic information is available, and (iii) the constraints in Eqs. (2.4) and (2.5) are satisfied. The control inputs u_1 and u_2 are chosen as

$$u_1 = \theta_1 f_l - \theta_1\lambda_1, \quad \text{and} \quad u_2 = \theta_1 f_r - \theta_1\lambda_3.$$

The following additional assumptions need to be made to establish the main result.

Assumption 12.1.

(A1) $\lambda_2(\dot{x}_{c2}, x_{c2}) = p^T\lambda(\dot{x}_{c2}, x_{c2})$, *where p is unknown.*

(A2) $$\frac{\partial\lambda_i(\dot{x}_{ci}, x_{c_i})}{\partial\,\dot{x}_{ci}} \neq 0 \quad \text{for } i = 1,3$$

(A3) $$\lim_{|x_{ci}|\to\infty} |\lambda_i(\dot{x}_{ci}, x_{ci})| = \infty, \quad \text{and} \quad \lim_{|\dot{x}_{ci}|\to\infty} |\lambda_i(\dot{x}_{ci}, x_{ci})| = \infty, \quad \text{for } i = 1,3$$

(A4) The origin $x = 0$ of the dynamical system

$$\lambda_i(\dot{x}, x) = 0 \qquad i = 1,3 \tag{12.1}$$

is globally asymptotically stable.

We make some comments about these assumptions before stating the main result.

1. Assumption (A2) implies that the coupling between the two degrees of freedom in the controlling unit, (i.e., in the finger) is through velocity. This is needed for generating a bounded input. If on the other hand, the controlling unit is coupled only by nonlinear springs, a stable controller can still be designed [13].
2. Assumption (A3) ensures that the stability result is global in character. In practice, this may not be realistic, but then a local stability result that is valid in a domain of attraction may suffice.
3. Assumption (A4) is needed to ensure that the zero dynamics of the system is asymptotically stable, which is standard in nonlinear control problems [15]. The somewhat nonstandard representation of Eq. (12.1) is used for the sake of convenience. The implicit function theorem ([15], p.404) can be used along with assumption (A2) to express Eq. (12.1) in a standard form of $\dot{x} = f(x)$.

From assumption (A1), Eq. (8.2) can be rewritten as

$$\begin{aligned}
&\ddot{x}_{c1} + \theta_2[\lambda_1(\dot{x}_{c1}, x_{c1}) - p^T\lambda(\dot{x}_{c2}, x_{c2})] = u_1\\
&\ddot{x}_{c2} - \theta_2[\lambda_1(\dot{x}_{c1}, x_{c1}) + \lambda_3(\dot{x}_{c2}, x_{c2}) + 2p^T\lambda(\dot{x}_{c2}, x_{c2})] = 0\\
&\ddot{x}_{c3} + \theta_2[\lambda_3(\dot{x}_{c3}, x_{c3}) - p^T\lambda(\dot{x}_{c2}, x_{c2})] = u_2\\
&\ddot{x}_3 - \theta_2[\lambda_3(\dot{x}_{c3}, x_{c3}) + p^T\lambda(\dot{x}_{c2}, x_{c2})] = 0
\end{aligned} \tag{12.2}$$

Our aim is to find an adaptive controller for the system in Eq. (12.2) which ensures that the state variables x_{c2} and x_3 and asymptotically track two prescribed reference trajectories $\overline{x}_{c2}(t)$ and $x^*(t)$ respectively, with all solutions of Eq. (12.2) remaining bounded. This is accomplished below.

Theorem 3. Let $\overline{x}_{c2}(t)$ and $x^*(t)$ be scalar bounded functions whose first three derivatives are bounded and accessible. Under assumptions (A1)-(A3), all solutions are globally bounded and

$$x_{c2}(t) \to \overline{x}_{c2}(t) \quad \text{and} \quad x_3(t) \to x^*(t) \text{ as } t \to \infty$$

if

$$u_1 = -\left(\frac{\partial \lambda_3}{\partial \dot{x}_{c3}} u_2 + W_1 + Z_1^T \hat{\xi} + d_1 e_1\right)\left(\frac{\partial \lambda_1}{\partial \dot{x}_{c1}}\right)^{-1}$$

$$u_2 = -\left(W_2 + Z_2^T \hat{\xi} + d_2 e_2\right)\left(\frac{\partial \lambda_3}{\partial \dot{x}_{c3}}\right)^{-1}$$

and

$$\dot{\hat{\eta}}_1 = (c_1 e_{c2} + (c_0+1)\,\dot{e}_{c2}) S_1$$
$$\dot{\hat{\eta}}_2 = (k_1 e_3 + (k_0+1)\,\dot{e}_3) S_2$$
$$\dot{\hat{p}} = [k_1 e_3 + (k_0+1)\,\dot{e}_3 + 2c_1 e_{c2} + 2(c_0+1)\,\dot{e}_{c2}]\lambda(\dot{x}_{c2}, x_{c2})$$
$$\dot{\hat{\xi}} = e_{c1} Z_1 + e_{c3} Z_2$$

where

$$\hat{\eta}_1 = [\hat{c}_0, \hat{c}_1, \hat{c}_2]^T, \quad \hat{\eta}_2 = [\hat{k}_0, \hat{k}_1, \hat{k}_2]^T, \quad \hat{p}, \text{ and } \hat{\xi} = [\hat{\xi}_1, \hat{\xi}_2^T]^T$$

are the estimates of

$$\eta_1 = \left[\frac{c_0}{\theta_2}, \frac{c_1}{\theta_2}, \frac{1}{\theta_2}\right]^T, \eta_2 = \left[\frac{k_0}{\theta_2}, \frac{k_1}{\theta_2}, \frac{1}{\theta_2}\right]^T, \; p, \text{ and } \xi = [\theta_2, \theta_2 p^T]^T \tag{12.3}$$

and

$$c_0 > 0,\; c_1 > 0,\; k_0 > 0,\; k_1 > 0,\; d_1 > 0,\; d_2 > 0,$$
$$e_{c2} = x_{c2} - \overline{x}_{c2}, \quad e_3 = x_3 - x^*,$$
$$e_{c1} = \lambda_1 + \lambda_3 + 2\hat{p}^T\lambda - \hat{c}_2 \dddot{\overline{x}}_{c2} + \hat{c}_1\,\dot{e}_{c2} + \hat{c}_0 e_{c2},$$
$$e_{c3} = \lambda_3 + \hat{p}^T\lambda - \hat{k}_2 \ddot{x}^* + \hat{k}_1\,\dot{e}_3 + \hat{k}_0 e_3$$
$$S_1 = [e_{c2}, \dot{e}_{c2}, -\dddot{\overline{x}}_{c2}]^T, \quad S_2 = [e_3, \dot{e}_3, -\ddot{x}^*]^T$$

$$Z_1 = \begin{bmatrix} -\frac{\partial \lambda_1}{\partial \dot{x}_{c1}}\lambda_1 - \frac{\partial \lambda_3}{\partial \dot{x}_{c3}}\lambda_3 + \left(\hat{c}_1 + 2\hat{p}^T \frac{\partial \lambda}{\partial \dot{x}_{c2}}\right)(\lambda_1 + \lambda_3) + \\ c_1 e_{c2} + (c_0+1)\,\dot{e}_{c2}\left(\frac{\partial \lambda_1}{\partial \dot{x}_{c1}} + \frac{\partial \lambda_3}{\partial \dot{x}_{c3}} + 2\left(\hat{c}_1 - 2\hat{p}^T \frac{\partial \lambda}{\partial \dot{x}_{c2}}\right)\right)\lambda \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} -\frac{\partial \lambda_3}{\partial \dot{x}_{c3}}\lambda_3 + \hat{p}^T \frac{\partial \lambda}{\partial \dot{x}_{c2}}(\lambda_1 + \lambda_3) + \hat{k}_1 \lambda_3 + k_1 e_3 + (k_0+1)\,\dot{e}_3 \\ \left(\frac{\partial \lambda_3}{\partial \dot{x}_{c3}} + 2\hat{p}^T \frac{\partial \lambda}{\partial \dot{x}_{c2}} + \hat{k}_1\right)\lambda \end{bmatrix}$$

$$W_1 = \frac{\partial \lambda_1}{\partial x_{c1}}\,\dot{x}_{c1} + \frac{\partial \lambda_3}{\partial x_{c3}}\,\dot{x}_{c3} + 2\,\dot{\hat{p}}^T\,\lambda + 2\hat{p}^T \frac{\partial \lambda}{\partial x_{c2}}\,\dot{x}_{c2} - \hat{c}_2 \overline{x}_{c2}^{(3)} - (\dot{\hat{c}}_2 + \hat{c}_1)\dddot{\overline{x}}_{c2} + (\dot{\hat{c}}_1 + \hat{c}_0)\,\dot{e}_{c2} + \dot{\hat{c}}_0\, e_{c2}$$

$$W_2 = \frac{\partial \lambda_3}{\partial x_{c3}}\,\dot{x}_{c3} + \dot{\hat{p}}^T\,\lambda + \hat{p}^T \frac{\partial \lambda}{\partial x_{c2}}\,\dot{x}_{c2} - \hat{k}_2 x^{*(3)} - \left(\dot{\hat{k}}_2 + \hat{k}_1\right)\ddot{x}^* + \left(\dot{\hat{k}}_1 + \hat{k}_0\right)\dot{e}_3 + \dot{\hat{k}}_0\, e_3.$$

Proof. We note that Eq. (12.2) has four degrees of freedom with only two inputs. From the second and fourth equation in Eq. (12.2), it could be viewed that x_{c1} and x_{c3} are 'external' variables affecting the motion of x_{c2} and x_3. Therefore, we define two reference trajectories $x_{c_{r1}}$ and $x_{c_{r3}}$ as

$$\dot{x}_{c_{r1}} = -\lambda_1 + \dot{x}_{c1} - \lambda_3 - 2\hat{p}^T\lambda + \hat{c}_2 \dddot{\overline{x}}_{c2} - \hat{c}_1\,\dot{e}_{c2} - \hat{c}_0 e_{c2}$$
$$\dot{x}_{c_{r3}} = -\lambda_3 + \dot{x}_{c3} - \hat{p}^T\lambda + \hat{k}_2 \ddot{x}^* - \hat{k}_1\,\dot{e}_3 - \hat{k}_0 e_3$$

This leads to errors e_{c1} and e_{c3} given by

$$\begin{aligned} e_{c1} &= \dot{x}_{c1} - \dot{x}_{c_{r1}} = \lambda_1 + \lambda_3 + 2\hat{p}^T\lambda - \hat{c}_2 \dddot{\overline{x}}_{c2} + \hat{c}_1\,\dot{e}_{c2} + \hat{c}_0 e_{c2} \\ e_{c3} &= \dot{x}_{c3} - \dot{x}_{c_{r3}} = \lambda_3 + \hat{p}^T\lambda - \hat{k}_2 \ddot{x}^* + \hat{k}_1\,\dot{e}_3 + \hat{k}_0 e_3 \end{aligned} \tag{12.4}$$

and parameter errors

$$\tilde{\eta}_1 = \hat{\eta}_1 - \eta_1, \quad \tilde{\eta}_2 = \hat{\eta}_2 - \eta_2, \quad \text{and} \quad \tilde{p} = \hat{p} - p.$$

Using the definitions in theorem 3, we can rewrite the second and fourth equations in Eq. (12.2) as

$$\ddot{e}_{c2} + c_1\,\dot{e}_{c2} + c_0 e_3 = \theta_2(e_{c1} - \tilde{\eta}_1^T S_1 - 2\tilde{p}^T\lambda)$$
$$\ddot{e}_3 + k_1\,\dot{e}_3 + k_0 e_3 = \theta_2(e_{c3} - \tilde{\eta}_2^T S_2 - \tilde{p}^T\lambda)$$

Furthermore, by defining

$$\overline{e}_{c2} = [e_{c2}, \dot{e}_{c2}]^T \qquad \overline{e}_3 = [e_3, \dot{e}_3]^T$$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -c_0 & -c_1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -k_0 & -k_1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

we have

$$\begin{aligned} \dot{\overline{e}}_{c2} &= A_1 \overline{e}_{c2} + B\theta_2(e_{c1} - \tilde{\eta}_1^T S_1 - 2\tilde{p}^T\lambda) \\ \dot{\overline{e}}_3 &= A_2 \overline{e}_3 + B\theta_2(e_{c3} - \tilde{\eta}_2^T S_2 - \tilde{p}^T\lambda). \end{aligned} \tag{12.5}$$

Now consider a Lyapunov function candidate given by

$$V = \frac{1}{2}\overline{e}_{c2}^T P_1 \overline{e}_{c2} + \frac{1}{2}\overline{e}_3^T P_2 \overline{e}_3 + \frac{1}{2}e_{c1}^2 + \frac{1}{2}e_{c3}^2 + \frac{1}{2}\theta_2 \tilde{\eta}_1{}^T \tilde{\eta}_1 + \tfrac{1}{2}\theta_2 \tilde{\eta}_2{}^T \tilde{\eta}_2 + \tfrac{1}{2}\theta_2 \tilde{p}^T \tilde{p} + \tfrac{1}{2}\tilde{\xi}^T \tilde{\xi}.$$

It is easy to show that by choosing $c_0 > 0,\ c_1 > 0,\ k_0 > 0$ and $k_1 > 0$, we obtain

$$P_1 = \begin{bmatrix} c_0^2 + c_1^2 + c_0 & c_1 \\ c_1 & c_0 + 1 \end{bmatrix} > 0, \quad P_2 = \begin{bmatrix} k_0^2 + k_1^2 + k_0 & k_1 \\ k_1 & k_0 + 1 \end{bmatrix} > 0$$

and hence

$$Q_1 = -(A_1^T P_1 + P_1 A_1) > 0$$

$$Q_2 = -(A_2^T P_2 + P_2 A_2) > 0.$$

Differentiating $V(x)$ along the trajectories of (12.2), we obtain

$$\dot{V} = -\frac{1}{2}\bar{e}_{c2}^T Q_1 \bar{e}_{c2} - \frac{1}{2}\bar{e}_3^T Q_2 \bar{e}_3 + \theta_2 \tilde{\eta}_1^T(\dot{\tilde{\eta}}_1 - \bar{e}_{c2}^T P_1 B S_1) + \theta_2 \tilde{\eta}_2^T(\dot{\tilde{\eta}}_2 - \bar{e}_3^T P_2 B S_2) + \theta_2 \tilde{p}^T[\dot{\tilde{p}} - (2\bar{e}_{c2}^T P_1 + \bar{e}_3^T P_2) B \lambda] + e_{c1}(\dot{e}_{c1} + \bar{e}_{c2}^T P_1 B \theta_2) + e_{c3}(\dot{e}_{c3} + \bar{e}_3^T P_2 B \theta_2) + \tilde{\xi}^T \dot{\tilde{\xi}}.$$

With the control laws u_1 and u_2, and the adaptive laws for $\hat{\eta}_1$, $\hat{\eta}_2$, $\hat{p}$ and $\hat{\xi}$ as in theorem 3, we finally obtain

$$\dot{V} = -\frac{1}{2}\bar{e}_{c2}^T Q_1 \bar{e}_{c2} - \frac{1}{2}\bar{e}_3^T Q_2 \bar{e}_3 - \frac{1}{2}d_1 e_{c1}^2 - \frac{1}{2}d_2 e_{c3}^2 \le 0$$

which implies that $\bar{e}_{c2}$, $\bar{e}_3$, e_{c1}, e_{c3}, $\tilde{\eta}_1$, $\tilde{\eta}_2$, $\tilde{p}$ and $\tilde{\xi}$ are bounded. The boundedness of $\bar{e}_{c2}$, $\overline{x}_{c2}$ and $\dot{\overline{x}}_{c2}$ proves x_{c2} and $\dot{x}_{c2}$ are bounded. Combining this with the fact that λ is a bounded function if its arguments are bounded further proves that $\lambda(x_{c2}, \dot{x}_{c2})$ is bounded. Also, from the Eq. (12.4), it can be easily seen that λ_1 and λ_3 are bounded, which leads to the conclusion that x_{c1}, $\dot{x}_{c1}$, x_{c3} and $\dot{x}_{c3}$ are bounded from assumption (A3). Moreover, the fact that $\bar{e}_{c2}$, $\bar{e}_3$, $\ddot{x}^*$ and $\ddot{\overline{x}}_{c2}$ are bounded implies that S_1 and S_2 are bounded. Combining all these together, we see, by Eq. (12.5), that $\dot{\bar{e}}_{c2}$ and $\dot{\bar{e}}_3$ are bounded. We therefore conclude $\bar{e}_{c2}(t) \to 0$ and $\bar{e}_3(t) \to 0$ as $t \to \infty$, which completes the proof. □

Remark:

(1) The unknown parameters θ_2 and p are estimated as η_1, η_2, p and ξ, which are defined in Eq. (12.3). Hence the asymptotic tracking is achieved at the price of overparametrization. The vector $[\theta_2, p^T]^T$ is overparametrized as $[\theta_2, p^T, \theta_2 p^T]^T$ in order to make the unknown parameters occur linearly. Such an overparametrization, however, is not uncommon in adaptive nonlinear control. The parameters η_1 and η_2 on the other hand are specific to the adaptive algorithm in our paper. They occur since the state variables x_{c1} and x_{c3} are used as fictitious inputs for controlling x_{c2} and x_3. By realizing this and noticing that the unknown parameter θ_2 is the coefficient of $\lambda_1(x_{c1}, \dot{x}_{c1})$ and $\lambda_3(x_{c3}, \dot{x}_{c3})$, an adaptive law can be generated following the standard adaptive controller design [11].

(2) As one can see from the design of the adaptive controller and the proof of stability, the assumption (A4) is not needed in Theorem 3. This is not surprising because the major concern in the tracking problem is to ensure that the states x_{c2} and x_3 follow the prescribed reference trajectories $\overline{x}_{c2}$ and x^*, respectively. As for the states x_{c1} and x_{c3}, the only requirement is that they be bounded. Assumption (A4) is needed only when asymptotic convergence of x_{c1} and x_{c3} to the origin is concerned. In [13], when $\lambda_1(\dot{x}, x) = (x^2 + 1)\ \dot{x} + 10x^3$, $\lambda_2(\dot{x}, x) = (p_0 x^2 + p_1)\ \dot{x} + p_2 x^3$, $x^* = 4\sin(0.2t - .1)$ and $x_{c2}^* = 0.9$, the simulation results indicated that the trajectory following was satisfied, and the object was retained in grasp at all times.

(3) Finally, we note that in the adaptive control problem considered here, we have assumed that (i) the finger parameters are known, (ii) both tactile and kinesthetic information is available, and (iii) the slip and crush constraints are satisfied. It is quite straight forward to extend the result to the case when (i) is not satisfied. Relaxation of (ii) requires the design of globally stable adaptive observers, which may be quite difficult to accomplish. Similarly, relaxing (iii) implies that a nonlinear control problem in the presence of magnitude constraints has to be solved, which is a nontrivial task.

13. Adaptive control of a general class of nonlinear systems with a triangular structure. The success of the adaptive controller in stabilizing the nonlinear system in Eq. (8.2) has led to the development of a general class of nonlinear systems with parametric uncertainties which can be globally stabilized and controlled [17]. These nonlinear systems can be divided into two categories, both of which possess a special triangular structure. These classes, denoted as $\mathcal{T}_1$ and $\mathcal{T}_2$, correspond to a set of first and second order nonlinear systems, and are described below. In all cases, u refers to a scalar external control input.

Definition 1 A system S is said to belong to $\mathcal{T}_1$ if it is described by

$$\text{(13.1)} \quad \begin{aligned} \dot{z}_i &= \gamma_i^0(z_1, ..., z_{i+1}) + \theta^T \gamma_i(z_1, ..., z_{i+1}), \qquad i = 1, ..., n-1 \\ \dot{z}_n &= \gamma_n^0(z) + \theta^T \gamma_n(z) + [\beta_0(z) + \theta^T \beta(z)]u \end{aligned}$$

where $z = [z_1, ..., z_n]^T$, $\theta = [\theta_1, \ldots, \theta_p]^T$ is a vector of unknown parameters belonging to a set $\Theta \subset \mathbb{R}^p$, and Θ is such that Eq. (13.1) is feedback equivalent to a controllable linear system for all $\theta \in \Theta$.

Definition 2 A system S is said to belong to $\mathcal{T}_{1s}$ if it belongs to $\mathcal{T}_1$ and in addition, for each $i = 1, \ldots, n-1$, there exists a unique $j_i \in [0, ..., p]$ such that

$$\left\{\begin{array}{lll} \dfrac{\partial \gamma_i^{j_i}}{\partial z_{i+1}} & \neq 0 & \text{for all } z, \text{ and} \\ \dfrac{\partial \gamma_i^{j}}{\partial z_{i+1}} & \equiv 0 & \forall\, j = 0, ..., p,\ j \neq j_i \end{array}\right. \tag{13.2}$$

and there is also a unique $j_n \in [0, ..., p]$ such that

$$\left\{\begin{array}{lll} \beta_{j_n}(z) & \neq 0 & \text{for all } z, \text{ and} \\ \beta_j(z) & \equiv 0 & \forall j = 0, \ldots, p,\ j \neq j_n. \end{array}\right. \tag{13.3}$$

Definition 3 A system S is said to belong to $\mathcal{T}_2$ if

$$\ddot{x}_i = \theta_i N_i(x_1, \dot{x}_1, .., x_{i+1}, \dot{x}_{i+1}) + p_i^T f_i(x_1, \dot{x}_1, .., x_i, \dot{x}_i), \quad i = 1, \ldots, n-1$$

$$\ddot{x}_n = \theta_n N_n(x) u + p_n^T f_n(x)$$

(13.4)

where $x = [x_1, \dot{x}_1, ..., x_n, \dot{x}_n]^T$, and $\theta = [\theta_1, ..., \theta_n]^T$ and $p_1, ..., p_n \in \mathbb{R}^p$ are unknown parameters.

Eq. (13.4) can be viewed as a direct extension of the nonlinear system considered in Eq. (8.2), with a single control input and $2n$ state variables, while Eqs. (13.1) and (13.2) possess a similar structure but correspond to a set of first-order nonlinear differential equations. The common feature to all these systems is the triangular structure in the differential equations. Even though the number of control inputs is significantly smaller than the number of state-variables, the triangular structure can be exploited to derive stability properties which hold in the large for these systems. The following theorems summarize these stability properties.

Theorem 4. Any system in $\mathcal{T}_1$ can be stabilized in a neighborhood Ω_z of the origin.

Theorem 5. The origin of a system in $\mathcal{T}_{1s}$ can be made globally stable.

Theorem 6. All states of the system in $\mathcal{T}_2$ will be globally bounded, and in addition $z_1(t)$ will asymptotically track a prescribed reference trajectory $z^*(t)$ if

(A1) the reference signal $x^*(t)$ and its first r derivatives, $x^{*(i)}(t),\ i = 1, ..r$, where r is the relative degree of the system in $\mathcal{T}_2$ are known and bounded for all $t \geq t_0$;

(A2) $N_i(\cdot)$ and $f_i(\cdot)$, $i = 1, ..., n-1$, are smooth functions, and for bounded $x_1, ..., x_i, \dot{x}_1, ..., \dot{x}_i$,

$$\lim_{|x_{i+1}| \to \infty} |N_i| = \infty, \quad \text{and} \quad \lim_{|\dot{x}_{i+1}| \to \infty} |N_i| = \infty, \quad \forall x \in R^{2n};$$

(A3) $N_n(x) \neq 0$ and either (i) $\dfrac{\partial N_i}{\partial\, \dot{x}_{i+1}} \neq 0$ or (ii) $\dfrac{\partial N_i}{\partial\, \dot{x}_{i+1}} = 0$, $\dfrac{\partial N_i}{\partial x_{i+1}} \neq 0,\ \forall x \in \mathbb{R}^{2n}$.

We refer the reader to [18] for the proofs and further details.

14. Summary. In this paper, we initiate a computational theory of haptics that focuses on the information processing aspects of manual exploration and manipulation. Concepts from mechanics, parameter identification, and control are combined to develop an analysis of haptics, which with suitable extensions provides a theoretical foundation for the design of haptic interfaces for virtual environments and teleoperation. The models developed here are based on the idea that passive compliance in human or robot fingers facilitates performance of contact tasks. From contact mechanics considerations as well as human studies, it is clear that addition of passive compliant fingerpads greatly enhances stability of grasp and manipulability. The interposition of this passive link between the object and the actively controlled rigid backing of the finger introduces complex dynamic relationships between the external forces exerted on the fingers and the contact forces imposed on the object. The deformation and motion of the object are dependent on the parameters governing the dynamic behavior of the object and the compliant fingerpads. In addition, to prevent slipping or crushing of the object, it is necessary to satisfy constraints on contact forces which are not directly controllable. Therefore, successful performance of grasping and manipulation of unknown objects requires either an explicit identification of the finger- and object-parameters followed by control algorithms tuned to the identified parameter values, or adaptive control algorithms that provide on-line compensating actions even when the system parameters are unknown.

In order to focus on the identification and control issues, we simplified the mechanics by employing lumped parameter models of the fingerpads and a generic compliant object with an internal degree of freedom. After deriving the dynamic and constraint equations for grasping and moving the object in a gravity environment, we showed that the presence of natural symmetries elegantly decouples the multivariable system into two single-input single-output problems corresponding to symmetric grasping and asymmetric motion. We then described the procedure to identify the constraint parameters, and applied well known results on identification of transfer functions to identify the dynamic parameters of the fingers and the object. Care was taken to satisfy the constraints arising from prevention of slipping and crushing of the object during the identification process. Identification procedures were also discussed for the case when the stiffness and viscosity properties of the object and the fingers are nonlinear. We presented adaptive control strategies for carrying out object manipulation for both linear and nonlinear models. It was shown that the approach developed to solve the adaptive control problem was applicable to a more general class of nonlinear systems possessing a certain triangular structure.

A conclusion that arises from the analyses carried out in this paper is that tactile information is of utmost importance to perform the contact tasks well with compliant fingerpads. Even when the fingerpad and object models are exact, tactile information reduces the order of the system to be identified, and provides access to all the states of the system during controlled manipulation. In reality, the fingerpad and object models considered only approximate the actual dynamic behavior. In order for the identification and control procedures to be robust, continuous monitoring of contact conditions with tactile sensors is absolutely necessary. In humans, specialized tactile mechanoreceptors enable detection of slipping of objects on the skin [9,12] and analogous robotic sensors have also been developed [19].

At present, humans perform dextrous manipulation much better than the best available robots. From biomechanical, neurophysiological, and psychophysical studies, it has been demonstrated that the mechanical properties of the fingerpads, and the continuous monitoring of the tasks through a wide variety of sensors whose output is processed by the brain and fed back to control the motor action of the muscles, contribute to the superior dexterity of the humans. It is possible that the identification and control algorithms presented here have their analogs in information acquisition and processing by the human peripheral and central nervous systems. Experimental investigations of human haptic performance designed to test specific hypotheses generated by the analysis presented here (perhaps with suitable extensions), would help resolve such issues. In robotics, we envision new robot end-effectors with compliant fingerpads which are richly innervated with tactile sensors of various types signalling a variety of information ranging from contact force distribution to skin vibrations caused by slipping of a grasped object. Interpretation of such sensor information and generation of appropriate motor actions requires a spatio-temporal mechanistic model of compliant fingerpads together with identification and control algorithms similar to the ones presented here. Especially in the case of haptic interfaces for virtual environments and teleoperation, where the human dynamics is directly coupled to that of the interface device, the stability of haptic interactions is seriously affected by time delays within the human haptic system as well as the by time taken for transmission and processing of information to and from the environment. For proper design and operation of such systems, theoretical analyses of the dynamics and control problem is essential. The purely temporal analysis presented here with lumped parameter models is only a first step towards building a computational theory of haptics that focusses on the information processing aspects independent of the hardware, be it robots or humans.

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