Virtual surgery simulation using a collocation-based method of finite spheres

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Abstract

The method of finite spheres using moving least squares interpolants and point collocation as the weighted residual scheme is applied to the development of a virtual reality based training system for laparoscopic surgical procedures. The localization of approximation and the lack of numerical integration results in very high computational speeds required for real time simulation with graphical and haptic feedback.

Keywords: Method of finite spheres; Meshless technique; Haptics; Medical simulation

1. Introduction

The objective of this paper is to illustrate how the method of finite spheres [1] may be applied to develop a laparoscopic surgical simulator which will enable the user to interact with three-dimensional computer models of biological tissues and organs in real time, using both visual and haptic sensory modalities. As minimally invasive surgery is gaining popularity, the need to train medical students and also to provide surgeons with appropriate computer tools to experiment with new surgical techniques, without having to use cadavers or animals, is becoming increasingly important.

The main challenge in real time virtual surgery is computational speed. For real time visual display an update rate of about 30 Hz is sufficient. To enable the user to interact with the computer models using the sense of touch we use a three degree-of-freedom haptic interface device called Phantom [1]. For stable real time simulation, the haptic loop requires to be updated at a rate of about 1 kHz.

A variety of simulation techniques, ranging from purely geometrical procedures without any physical basis to spring–mass–dashpot-based models, are found in the literature (see reference [2] for a summary of the existing techniques). Although the finite element technique [3] is a physically based procedure, it is computationally very slow since the entire domain needs to be meshed and numerical integration has to be performed.

In this paper we develop a specialized version of the method of finite spheres based on moving least squares interpolants and point collocation for the purpose of real time surgical simulations.

2. The numerical scheme

In our technique, N nodal points are sprinkled around the surgical tool tip (see Fig. 1). Moving least squares interpolants

$$h_J(\mathbf{x}) = W_J(\mathbf{x})\mathbf{P}(\mathbf{x})^{\mathrm{T}}\mathbf{A}^{-1}(\mathbf{x})\mathbf{P}(\mathbf{x}_J) \qquad J = 1, \ldots, N \tag{1}$$

are used to generate the local finite dimensional approximation spaces.

In Eq. (1) $\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{N} W_I(\mathbf{x})\mathbf{P}(\mathbf{x}_I)\mathbf{P}(\mathbf{x}_I)^{\mathrm{T}}$. The vector $\mathbf{P}(\mathbf{x})$ contains polynomials ensuring consistency up to a desired order (in our implementation we have ensured consistency up to degree one). W_J is a compactly supported radial weighting function at node J (which we have chosen as a quartic spline function).

We assume linear elastic tissue behavior. A point collocation technique is used to generate the discrete equations

$$\mathbf{KU} = \mathbf{f} \tag{2}$$

where $\mathbf{K}$ is the stiffness matrix and $\mathbf{f}$ is the vector con-

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[1] Developed by SensAble Technologies, Inc.

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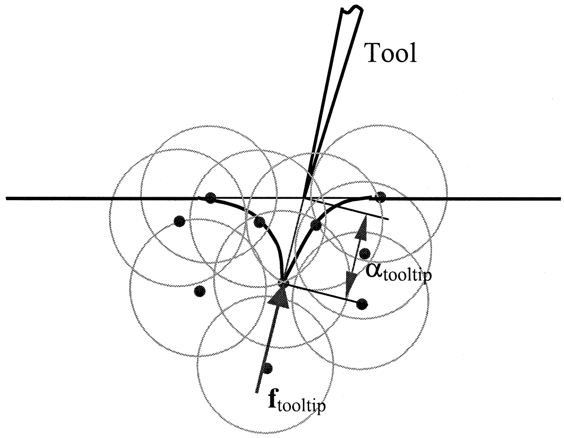

Fig. 1. A schematic showing the distribution of nodal points around a surgical tool tip.

taining nodal loads. We note here the stiffness matrix $\mathbf{K}$ is nonsymmetric, but banded.

For the purpose of surgical simulation, the tool tip may be modeled as having point interaction with the tissue (see Fig. 1). A node is placed at the tool tip and all other nodes are placed such that their spheres do not intersect the node at the tool tip (or do so only minimally to ensure the invertibility of $\mathbf{A}(\boldsymbol{x})$). The node at the tool tip bears the applied displacement, $\mathbf{U}_{\text{tool tip}}$ at the tool tip.

The stiffness matrix in Eq. (1) may be partitioned as

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{aa} & \mathbf{K}_{ab} \\ \mathbf{K}_{ba} & \mathbf{K}_{bb} \end{bmatrix} \tag{3}$$

corresponding to a partitioning of the vector of nodal parameters as $\mathbf{U} = \left[\mathbf{U}_{\text{tool tip}} \; \mathbf{U}_b\right]^{\mathrm{T}}$ where $\mathbf{U}_b$ is the vector of nodal unknowns which maybe obtained as $\mathbf{U}_b = -\mathbf{K}_{bb}^{-1}\mathbf{K}_{ba}\mathbf{U}_{\text{toool tip}}$. The reaction force to be delivered to the haptic interface device is obtained as $\mathbf{f}_{\text{tool tip}} = \mathbf{K}_{aa}\mathbf{U}_{\text{tool tip}} + \mathbf{K}_{ab}\mathbf{U}_b$.

3. Simulation demonstration

Fig. 2 shows the deformation field computed using the technique described in the previous section when a tool interacts with a hemispherical object. Linear elastic tissue behavior was assumed. The undeformed surface as also the deformation obtained using ADINA with a finite element discretization of the object are presented for reference.

The point collocation based method of finite spheres provides reasonable deformation fields near the tool tip but the errors are quite high further away. This technique is however very fast. Computational rate of about 100 Hz is obtainable for the example shown in Fig. 2 when 34 spheres are used for discretization. Real time rendering rates of about 1 kHz is obtained using a force extrapolation technique (refer to [2] for details).

References

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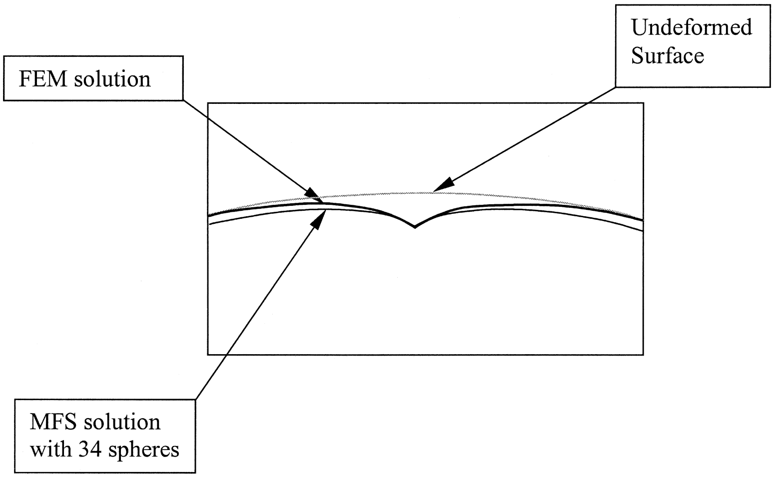

Fig. 2. The deformation field obtained when MFS is used for the simulation of a surgical tool tip interacting with a hemispherical object is shown. The undeformed surface and the deformation field obtained using a finite element discretization are also shown.