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A Meshless Numerical Technique for Physically Based Real Time Medical Simulations

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Abstract. This work introduces, for the first time, a meshless modeling technique, the method of finite spheres, for physically based, real time rendering of soft tissues in medical simulations. The technique is conceptually similar to the traditional finite element techniques. However, while the finite element techniques requires a slow mesh generation process, this new technique has significant potential for multimodal medical simulations of the future since it does not use a mesh. Several examples are presented showing the effectiveness of the scheme.

1. Introduction

In this paper, we present a novel "meshless" numerical scheme for computations underlying virtual reality based medical simulations. This technique enables the user to interact with physically based tissue and organ models in real time, using both visual and haptic sensory modalities. Of special interest are surgical simulations involving contact interactions between long slender tools and deformable organs. Laparoscopic surgery is a particular example of such a procedure.

The success of a laparoscopic surgeon depends heavily on his or her training. An efficient laparoscopic surgery simulator will not only result in customized practice environments for medical students, but will also reduce the use of animals and cadavers that are currently used for such training.

An important issue in medical simulation is the modeling of soft tissues. From a purely mechanistic viewpoint, soft tissues exhibit complex material properties [1]. They are nonlinear, anisotropic, viscoelastic and nonhomogeneous (usually layered). Moreover, soft tissues deform considerably under the application of relatively small loads. In addition, it is quite difficult to obtain *in vivo* material properties of living tissues. Therefore a challenging task is to develop efficient models for living tissues so that realistic simulation of tool-tissue interaction can be performed in real time.

For real time visual display an update rate of 30 Hz is sufficient. For haptic display, we use the Phantom haptic interface device (SensAble Technologies, Inc). For stable simulation, the haptic loop requires an update rate of about 1kHz. This imposes severe restrictions on the complexity of the models that can be rendered haptically. Therefore simulation speed is the prime consideration.

Various techniques can be found in literature for the simulation and display of deformable objects. These techniques can be categorized into two main approaches: "geometrically based" approaches and "physically based" approaches [2]. The "geometrically based" modeling approaches, such as Bezier/B-spline based procedures and free form deformation techniques, do not account for the physics of deformation, but are simpler to implement.

In contrast, the "physically based" approaches, such as the lumped parameter spring-mass-damper models [3] attempt to model the underlying physics. These models are simple and computationally very efficient. However, the construction of an optimal network of springs in 3D is a complicated process. Moreover, under certain conditions, mass-spring systems may become oscillatory or even go unstable during simulation.

The finite element technique is widely used in engineering analysis for the simulation of deformable objects [4]. Several researchers have applied finite element techniques for real time surgical simulations [5, 6]. However, finite element techniques suffer from certain drawbacks in real time simulations. First, the contact between tool and tissues must occur at nodal points. Therefore, to prevent loss of resolution, the density of nodal points should be sufficiently high. This requires extensive memory resources and high computational overhead. Second, cutting or tearing requires an expensive remeshing process during simulation. This means precomputed data of the object becomes, at least locally, useless and all data must be computed in real time. The computation time increases approximately as the cube of the number of nodal unknowns. This poses significant obstacles in real time applications, given the high rate of force updates required.

A solution to the problems that are faced by the finite element techniques is to use a numerical technique that does not use a mesh. The method of finite spheres (MFS) is one such "meshless" computational technique [7]. In this paper we present a specialized version of the method of finite spheres for the purpose of real time medical simulations. Nodal points are sprinkled around the surgical tool tip (not the entire domain) and the interpolation is performed by functions that are nonzero only on spheres surrounding the nodes. A point collocation technique is used to generate the discrete equations that are solved in real time.

The localization provided by the finite influence zones of the nodes as well as the elimination of numerical integration results in a highly accelerated numerical scheme. The flexibility in the placement of nodes allows complex operations like cutting to take place relatively easily. In addition, since the differential equations governing the tool tissue interactions are being solved in the vicinity of the tool tip, the solution procedure is physically based.

In the next section we briefly introduce the numerical technique. In section 3 we discuss numerical implementation issues and finally present some examples in section 4.

2. The numerical scheme

In this section we briefly introduce the theory behind the method of finite spheres (refer to [7] for details). In this technique, we distribute nodal points around the surgical tool tip and define spherical "influence zones" around each node (see Figure 1). The approximation u_h of a variable u (e.g. displacement), using 'N' spheres, may be written as

$$u_h(\mathbf{x}) = \sum_{J=1}^{N} h_J(\mathbf{x})\alpha_J \tag{1}$$

where α_J is the nodal unknown at node J. The nodal shape function $h_J(\mathbf{x})$ at node J is generated using a moving least squares technique [8]

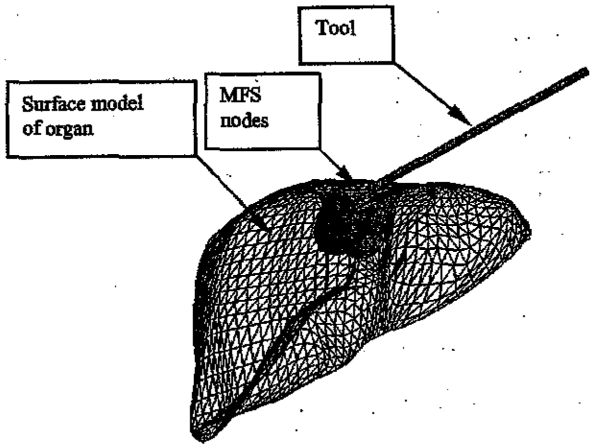

Figure 1. A laparoscopic surgical tool interacting with the surface model of a liver. MFS nodes are distributed in the vicinity of the tool tip to obtain a localized discretization.

$$\mathbf{h}_J(\mathbf{x}) = \mathbf{W}_J(\mathbf{x})\mathbf{P}(\mathbf{x})^T\mathbf{A}^{-1}(\mathbf{x})\mathbf{P}(\mathbf{x}_J) \qquad J=1,\ldots,N \tag{2}$$

where

$$\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{N} \mathbf{W}_I(\mathbf{x})\mathbf{P}(\mathbf{x}_I)\mathbf{P}(\mathbf{x}_I)^T . \tag{3}$$

The vector $\mathbf{P}(\mathbf{x})$ contains polynomials ensuring consistency up to a desired order (in our implementation we have chosen $\mathbf{P}(\mathbf{x}) = \{1,x,y,z\}^T$ to ensure a first order accurate scheme in 3D, similar to bilinear finite elements). W_J is a compactly supported radial weighting function at node J (which we have chosen as a quartic spline function).

We assume linear elastic tissue behavior. A point collocation technique is used to generate the discrete equations

$$\mathbf{KU} = \mathbf{f} \tag{4}$$

where $\mathbf{K}$ is the stiffness matrix and $\mathbf{f}$ is the vector containing nodal loads.

In surgical simulation, the tool tip may be modeled as having point interaction with the tissue (see Figure 2). A node is placed at the tool tip and all other nodes are placed such that their spheres do not intersect the node at the tool tip (or do so only minimally to ensure the invertibility of $\mathbf{A}(\mathbf{x})$ in Eq (3)). The nodal displacement at the tool tip is equal to the applied displacement, $\mathbf{U}_{tooltip}$.

The stiffness matrix in Eq (3) may be partitioned as

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{aa} & \mathbf{K}_{ab} \\ \mathbf{K}_{ba} & \mathbf{K}_{bb} \end{bmatrix} \tag{5}$$

corresponding to a partitioning of the vector of nodal parameters as $\mathbf{U} = [\mathbf{U}_{tooltip} \quad \mathbf{U}_b]^T$ where $\mathbf{U}_b$ is the vector of nodal unknowns which maybe obtained as $\mathbf{U}_b = -\mathbf{K}_{bb}^{-1}\mathbf{K}_{ba}\mathbf{U}_{tooltip}$. The reaction force to be delivered to the haptic interface device is obtained as $\mathbf{f}_{tooltip} = \mathbf{K}_{aa}\mathbf{U}_{tooltip} + \mathbf{K}_{ab}\mathbf{U}_b$.

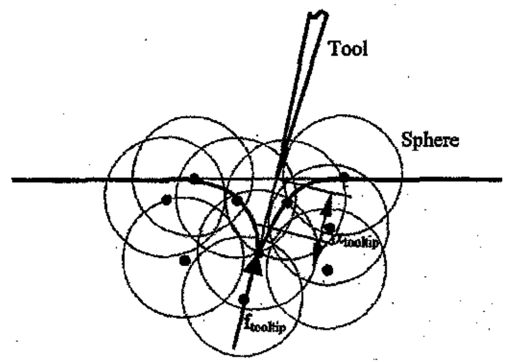

Figure 2. Placement of nodes at the tool tip. $\alpha_{tooltip}$ and $f_{tooltip}$ are the prescribed displacement and reaction force at the tool tip, respectively.

3. Real time issues

To simulate tissue palpation through a tool, collision detection, placement of the nodal points and computation of organ surface deformation and tool tip reaction force have to be performed in real time. The organ model is usually a polygon (triangular) based surface model. We use a fast collision detection algorithm developed by Ho et. al. [9] where we establish a hierarchical database of geometric primitives, with each primitive having pointers to neighboring primitives. The collision detection time is independent of the total number of polygons in the model and the process is, of course, real time (i.e. 1kHz and higher).

As soon as a collision point is detected, MFS nodes are sprinkled around the tool tip, both on the surface of the organ model as well as inside. This is a computationally intensive process. Therefore the location of the nodes relative to the tool tip is defined offline but the computation of tissue deformation is performed in real time. Another important issue is the choice of the radii of the spheres. While spheres with larger radii provide greater covering for fewer spheres, they increase the bandwidth of the stiffness matrix and also result in coarser approximation. Details regarding these issues may be found in [10].

4. Simulation demonstration

We have implemented the point collocation based method of finite spheres for real time simulation and display of deformation and tool tip reaction force for certain simple 3D geometries such as a hemisphere (see Figure 3) and a liver model (see Figure 4). In both cases, linear elastic tissue behavior has been assumed. These example problems illustrate the applicability of the new scheme proposed here to the simulation of tissue palpation.

In Figure 3 we compare the displacement solution results obtained for the hemisphere problem using the method of finite spheres (using 34 nodes) and the finite element technique (using the commercial finite element package ADINA and 27 noded volumetric brick elements). In the vicinity of the tool tip the displacement profile computed using the method of finite spheres is observed to match the much slower finite element solution and hence is quite accurate. However, increasing divergence is observed away from the tool tip. For the purpose of surgical simulation, this displacement profile based on MFS may be admissible.

The point collocation based method of finite spheres is however very fast. We were able to achieve a computational rate of about 100Hz for the example shown in Figure 2 when 34 spheres were used for discretization. Real time rendering rates of about 1 kHz was then obtained using a force extrapolation technique (refer to [10] for details).

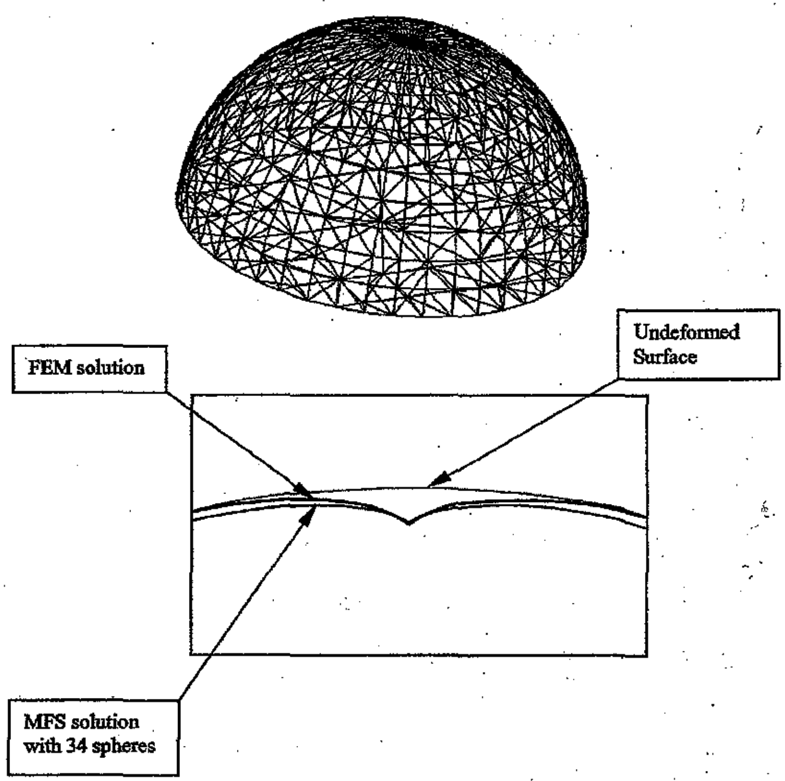

Figure 3. The deformation field obtained when MFS is used for the simulation of a surgical tool tip interacting with a hemispherical object is shown. The undeformed surface and the deformation field obtained using a finite element discretization are also shown.

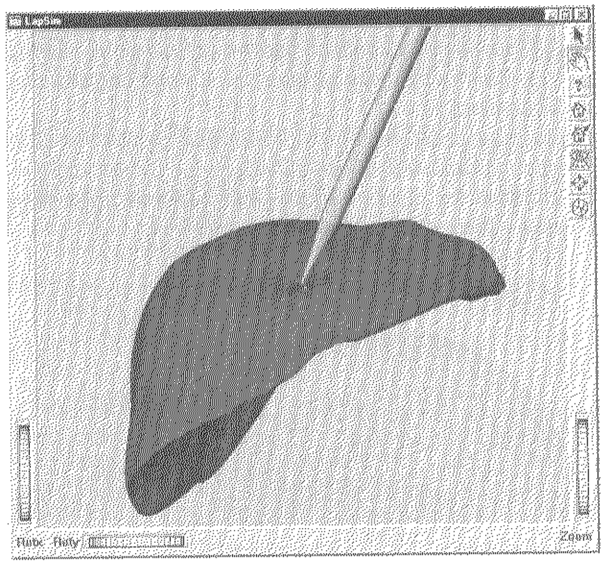

Figure 4. A snapshot of the laparoscopic surgical simulator LapSim showing a surgical tool interacting with a liver model.

References

[1] S. De and M. A. Srinivasan, Thin Walled Models for Haptic and Graphical Rendering of Soft Tissues in Surgical Simulation, *Proceeding of MMVR 7 Conference* (1999) 94-99.

[2] D. Terzopoulos, J. Platt, A. Barr and K. Fleischer, Elastically Deformable Models, *Computer Graphics* **21**(4) (1987) 205-214.

[3] D. Terzopoulos and K. Waters, Physically Based Facial Modeling, Analysis and Animation, *Journal of Visualization and Computer Animation* **1** (1990) 73-80.

[4] K. J. Bathe, Finite Element Procedures, Prentice Hall, 1996.

[5] M. Bro-Nielsen and S. Cotin, Real Time Volumetric Deformable Models for Surgery Simulation Using Finite Elements and Condensation, *Proceeding of Computer Graphics Forum, Eurographics 96* (1996) 57-66.

[6] S. Cotin, H. Delingette, J. M. Clement, V. Tassetti, J. Marescaux and N. Ayache, Geometric and Physical Representations for a Simulator of Hepatic Surgery, *Proceedings of MMVR 4 Conference* (1996)139-151.

[7] S. De and K. J. Bathe, The Method of Finite Spheres. Computational Mechanics, **25** (2000) 329-345.

[8] S.De and K. J.Bathe, Towards an Efficient Meshless Computational Technique: The Method of Finite Spheres, to appear in *Engineering Computations.*

[9] C. Ho, C. Basdogan and M. A. Srinivasan, Efficient Point-Based Rendering Techniques for Haptic Display of Virtual Objects, *Presence* **8** (1999) 477-491.

[10] S. De, J. Kim and M. A. Srinivasan, The Method of Finite Spheres in Real Time Multimodal Medical Simulations, to appear.